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EVALUATION OF FUNCTIONS WITH MATRIX ARGUMENTS IN A COMPUTATIONAL MATHEMATICS SYSTEM

Abstract. Functions with matrix arguments represent a mathematical generalization of functions of numerical arguments. Of particular interest is the determination of functions of matrices and the calculation of their values, primarily in connection with the solution of applied problems. It is possible to minimise the computational complexity, which is related not only to the increase in the order of the matrix but also to the presence of simple and multiple roots of the matrix's characteristic polynomial, by performing such computations in modern computer mathematics software packages. The present article develops code for computing a function with a matrix argument, in the case of simple roots of the matrix's characteristic polynomial, in the Maple system.

Keywords: matrix argument, characteristic roots, Lagrange–Sylvester interpolation polynomial, matrix spectrum, minimal polynomial.

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Introduction. In matrix analysis, particular attention is paid to matrix functions. This is due both to their use in fundamental research and to their numerous applications – in control theory, quantum mechanics, and electrical engineering. Let us consider the definition of matrix functions and the calculation of their values.

Conditions and methods of the study. Let A be a n square matrix:

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

$f(\lambda)$ is an arbitrary function of the scalar argument λ. The following considerations are to be made $f(\lambda)$ with matrix values, that is, the objective of this study is to define the function of matrices. $f(A)$. As is well known, the simplest functions of matrices are polynomials. The following investigation will consider the $f(\lambda)$ with matrix values. The objective is to define a function of matrices $f(A)$. It is a recognised fact that the simplest functions of matrices are polynomials. Thus, if $f(\lambda)$ is of the form:

$$f(\lambda) = \gamma_0 \lambda^l + \gamma_1 \lambda^{l-1} + ... + \gamma_l ,$$

then, substituting $\lambda = A$, we get:

$$f(A) = \gamma_0 A^l + \gamma_1 A^{l-1} + ... + \gamma_l E.$$

Given this specific case, it is interesting to consider the general case. Let $\psi_A(\lambda)$ be the minimal polynomial of the matrix A:

$$\psi_A(\lambda) = (\lambda - \lambda_1)^{m_1} (\lambda - \lambda_2)^{m_2} ... (\lambda - \lambda_s)^{m_в} , \quad (1)$$

degrees of $m : m = \deg \psi_A(\lambda) = \sum_{k=1}^{S} m_k$, where $\lambda_1, \lambda_2 ..., \lambda_S -$ eigenvalues of a matrix A, and various: $\lambda_1 \neq \lambda_2 \neq ... \neq \lambda_S$, $\sigma(A) = \{\lambda_1, .\lambda_2, ..., \lambda_s\}$.

Suppose that the polynomials $g(\lambda)$ and $h(\lambda)$ are equal, i.e.

$$g(A) = h(A). \quad (2)$$

Then their difference will be equal to 0: $g(\lambda) - h(\lambda) = 0$. Denoting the difference as $d(\lambda) = g(\lambda) - h(\lambda)$, we find that $d(\lambda)$ is a nilpolynomial for the matrix A. Then the difference $d(\lambda)$ is divisible by $\psi_A(\lambda)$ without a remainder, which can be written as:

$$g(\lambda) = h(\lambda) \ (\mathrm{mod} \psi(\lambda)). \quad (3)$$

Hence, by virtue of (1)

$$d(\lambda_k) = 0,\ d'(\lambda_k) = 0, ..., d^{(m_k - 1)}(\lambda_k) = 0 \ (k = 1,2,...,s),$$

i.e.

$$g(\lambda_k) = h(\lambda_k), g'(\lambda_k) = h'(\lambda_k), .., g^{(m_k - 1)}(\lambda_k) = h^{(m_k - 1)}(\lambda_k) \ (k = 1,2,...,s). \quad (4)$$

The values of function $f(\lambda)$ on the spectrum of matrix A are the m numbers:

$$f(\lambda_k), f'(\lambda_k), ..., f^{(m_k - 1)}(\lambda_k) \ (k = 1,2,...,s). \quad (5)$$

It is asserted that the function $f(\lambda)$ is defined on the spectrum of the matrix A. The system A is designated as a system of values of the function $f(\lambda)$ on the spectrum of the matrix A. In the absence of definition of the function $f(\lambda)$ on the spectrum of the matrix A, the function A is also undefined. In the context of the system (5), the following notation is employed: $f(\Lambda_A)$.. [1–3].

So the function $f(\lambda)=e^{\lambda}$ is defined on the matrix spectrum:

$$A=\begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The minimal polynomial of the matrix $\psi_A(\lambda)=(\lambda+1)^2(\lambda-2)$ is defined as having two roots $\lambda_1=-1$, with a multiplier $m_1=2$ and, as well as another root $\lambda_2=2$ with a multiplier $m_2=1$. The function $f(\lambda)=e^{\lambda}$ is defined by the set of values and its derivative.

to $\lambda_1=-1$: $f(\lambda_1)=e^{\lambda_1}$, $f(-1)=e^{-1}=\frac{1}{e}$; $f'(\lambda_1)=\lambda_1 e^{\lambda_1}$,

$f'(-1)=-e^{-1}=-\frac{1}{e}$;

to $\lambda_2=2$: $f(\lambda_2)=e^{\lambda_2}$, $f(2)=e^2$.

This function $f(\lambda)=e^{\lambda}$ is defined on the spectrum of the matrix A, and the set of values $f(\Lambda_A)$. is as follows: $f(\Lambda_A)=\left\{\frac{1}{e},-\frac{1}{e},e^2\right\}$.

The same cannot be said of the function $f(\lambda)=\frac{1}{\lambda}$ on the matrix spectrum $B: B=\begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}$. The minimal polynomial of the matrix B: $\psi_B(\lambda)=\lambda^2$ has one root $\lambda=0$ with a multiplier $m=2$. Function values $f(\lambda)=\frac{1}{\lambda}$ when $\lambda=0$ is not specified. Therefore, the function $f(\lambda)=\frac{1}{\lambda}$ is undefined on the matrix spectrum B. Therefore, the function of the matrix is undefined $f(B)$.

It has been demonstrated that the function $f(\lambda)=\frac{1}{\lambda}$ on the matrix spectrum $B: B=\begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}$ does not hold the same validity. The minimal polynomial of the matrix B: $\psi_B(\lambda)=\lambda^2$ is defined as having one root, $\lambda=0$ with a multiplier

$m=2$ In the absence of specification regarding the function values $f(\lambda)=\dfrac{1}{\lambda}$ the following function values $\lambda=0$ are to be employed: It can thus be concluded that the function $f(\lambda)=\dfrac{1}{\lambda}$ is undefined on the matrix spectrum B. In consequence, the function $f(B)$ of the matrix is undefined.

It is evident that for $f(A)$ to be comprehensible, it is sufficient that the function $f(\lambda)$ be defined at the characteristic points $\lambda_1,\lambda_2,...,\lambda_m$. In the event that $\psi_A(\lambda)$ possesses multiple roots, then the derivatives $f(\lambda)$ up to a certain order must be defined at specific characteristic points. From the equations (3), it is evident that the polynomials $g(\lambda)$ and $h(\lambda)$ possess identical eigenvalues within the spectrum of the matrix $A: g(\Lambda_A)=h(\Lambda_A)$.

The converse is likewise valid: if (3) follows (2), then (2) holds.

It can thus be concluded that, given a matrix A, the values of the polynomial $g(\lambda)$ on the spectrum of matrix A are unique in determining the matrix $g(A)$. It is evident that all polynomials $g(\lambda)$ that possess identical values on the spectrum of the matrix A also possess identical matrix values $g(A)$.. This assertion is applicable to the general case, that is to say, the values of the function $f(\lambda)$ on the spectrum of the matrix A must completely determine $f(A)$. Inbrief, all evident functions $f(\lambda)$ exhibit equivalent values within the spectrum of the matrix A. This observation persists when the matrix is considered as the argument. It is evident that in order to define the function of the matrices $f(A)$, it is sufficient to find a polynomial $g(\lambda)$ with the same values on the spectrum of the matrix A as $f(\lambda)$. In this particular instance, the following elements are at stake:

$$f(A)=g(A).$$

The following definition is thus proposed: In the event of a function $f(\lambda)$ being defined on the spectrum of the matrix A, it can be concluded that $f(A)=g(A)$, is a polynomial whose matrix spectrum contains the same values as $f(\lambda)$:

$$g(\Lambda_A)=h(\Lambda_A).$$

It is evident that there is at least one polynomial of degree $g(\lambda)$ whose eigenvalues in the spectrum of the matrix A coincide with those of $f(\lambda)$, we can speak of an infinite number of such polynomials. Among these, there is a unique polynomial of degree $r_A(\lambda)$ which has a degree less than m and such that:

$$f(\Lambda_A)=r(\Lambda_A).$$

The ensuing interpolation conditions are applicable to the polynomial: $r_A(\lambda)$:

$$r(\lambda_k) = f(\lambda_k), r'(\lambda_k) = f'(\lambda_k), ..., r^{(m_k-1)}(\lambda_k) = f^{(m_k-1)}(\lambda_k) \ (k=1,2,...,s) \quad (6)$$

which uniquely determine it. The polynomial $r_A(\lambda)$ is called the Lagrange–Sylvester interpolation polynomial for the function $f(\lambda)$ on the spectrum of the matrix A. [2]

It can thus be concluded that, in the event of the function $f(\lambda)$ being defined on the spectrum of the matrix $f(\lambda)$, the Lagrange–Sylvester interpolation polynomial corresponding to $r_A(\lambda)$ is therefore its inverse.

$$f(A) = r(A).$$

In the event of $r_A(\lambda)$ the Lagrange–Sylvester interpolation polynomial for the function $f(\lambda)$ being considered on the spectrum of the matrix A, the following equation is obtained:

$$f(A) = r(A) = \{r(A_1), r(A_2), ..., r(A_u)\}. \quad (7)$$

It is widely accepted that the minimal polynomial $\psi_A(\lambda)$ for A is regarded as the nullifying polynomial for each of the matrices $A_1, A_2, ..., A_u$ from the given equation $A_1, A_2, ..., A_u$.

$$f(\Lambda_{A_1}) = r(\Lambda_{A_1}), ..., f(\Lambda_{A_u}) = r(\Lambda_{A_u}).$$

It is

$$f(A_1) = r(A_1), ..., f(A_u) = r(A_u),$$

The following is the representation of equality (7):

$$f(A) = \{f(A_1), f(A_2), ..., f(A_u)\}. \quad (8)$$

In order to facilitate an adequate analysis of this particular problem, it is necessary to establish certain fundamental concepts. Thus, for a given scalar function $f(\lambda)$ and matrix A, the construction of a polynomial $r_A(\lambda)$ should be undertaken, such that the values $f(\lambda)$ and $r_A(\lambda)$ coincide on the matrix spectrum.

$$f(\Lambda_A) = r(\Lambda_A)$$

1) A case where the characteristic equation $|\lambda E - A| = 0$ has no multiple roots. Let $\lambda_1, \lambda_2, ..., \lambda_n$ be the roots of this equation, which are the characteristic values of the matrix A. Then

$$\psi_A(\lambda) = |\lambda E - A| = (\lambda - \lambda_1)(\lambda - \lambda_2)...(\lambda - \lambda_n),$$

and condition (6) is written as follows:

$$r(\lambda_k) = f(\lambda_k)\ k = 1,2,...,n).$$

In this case $r_A(\lambda)$ is the standard Lagrange interpolation polynomial for the function $f(\lambda)$ at points $\lambda_1, \lambda_2, ..., \lambda_n$:

$$r_A(\lambda) = \sum_{k=1}^{n} \frac{(\lambda - \lambda_1)...(\lambda - \lambda_{k-1})(\lambda - \lambda_{k+1})...(\lambda - \lambda_n)}{(\lambda_k - \lambda_1)...(\lambda_k - \lambda_{k-1})(\lambda_k - \lambda_{k+1})...(\lambda_k - \lambda_n)} f(\lambda_k),$$

Thereafter:

$$f(A) = r(A) = \sum_{k=1}^{n} \frac{(A - \lambda_1 E)...(A - \lambda_{k-1})(A - \lambda_{k+1} E)...(A - \lambda_n E)}{(\lambda_k - \lambda_1)...(\lambda_k - \lambda_{k-1})(\lambda_k - \lambda_{k+1})...(\lambda_k - \lambda_n)} f(\lambda_k).$$

2) Roots of a characteristic polynomial may be multiple while those of a minimal polynomial are simple: $\psi(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)...(\lambda - \lambda_m).$

In this instance, as was the case in Case 1, it can be demonstrated that all exponents of m_k in are equivalent to one. The following equation (6) can be written in its various forms:

$$r(\lambda_k) = f(\lambda_k)\ k = 1,2,...,m).$$

The number $r_A(\lambda)$ is represented in the form of a standard Lagrange interpolation polynomial.

$$f(A) = r(A) = \sum_{k=1}^{m} \frac{(A - \lambda_1 E)...(A - \lambda_{k-1})(A - \lambda_{k+1} E)...(A - \lambda_m E)}{(\lambda_k - \lambda_1)...(\lambda_k - \lambda_{k-1})(\lambda_k - \lambda_{k+1})...(\lambda_k - \lambda_m)} f(\lambda_k).$$

3) This is the general case.

$$\psi(\lambda) = (\lambda - \lambda_1)^{m_1}(\lambda - \lambda_2)^{m_2}...(\lambda - \lambda_s)^{m_s}\ m_1 + m_2 + ... + m_s = m).$$

The following hypothesis is put forward: $\frac{r(\lambda)}{\psi(\lambda)}$ is a proper rational function, and can be expressed as the sum of simple fractions:

$$\frac{r(\lambda)}{\psi(\lambda)} = \sum_{k=1}^{s} \left[\frac{\alpha_{k1}}{(\lambda - \lambda_k)^{m_k}} + \frac{\alpha_{k2}}{(\lambda - \lambda_k)^{m_k - 1}} + ... + \frac{\alpha_{k,m_k}}{\lambda - \lambda_k} \right], \quad (9)$$

where $\alpha_{kj} \, (j = 1,2,...,m_k; k = 1,2,...,s) -$ some numbers.

Let's identify the numerators of simple fractions α_{kj} in (9). In order to accomplish this task, it is necessary to multiply both sides of the final equation by $(\lambda - \lambda_k)^m{}_k$. Let us introduce the following notation for the resulting expression $\psi_k(\lambda)$ of the polynomial $\frac{\psi(\lambda)}{(\lambda - \lambda_k)^{m_k}}$. The result is:

$$\frac{r(\lambda)}{\psi_k(\lambda)} = \alpha_{k_1} + \alpha_{k_2}(\lambda - \lambda_k) + ... + \alpha_{km_k}(\lambda - \lambda_k)^{m_k - 1} + (\lambda - \lambda_k)^{m_k}\delta(\lambda)$$
$$(k = 1,2,...,s), \tag{10}$$

where $\delta(\lambda) -$ a rational function that is regular at $\lambda = \lambda_k$. From this, we obtain the following formulas (11):

$$\left.\begin{aligned} \alpha_{k_1} &= \left[\frac{r[\lambda]}{\psi_k(\lambda)}\right]_{\lambda=\lambda_k}, \\ \alpha_{k_2} &= \left[\frac{r(\lambda)}{\psi_k(\lambda)}\right]'_{\lambda=\lambda_k} = r(\lambda_k)\left[\frac{1}{\psi_k(\lambda)}\right]'_{\lambda=\lambda_k} + r'(\lambda_k)\frac{1}{\psi_k(\lambda_k)}, ...(k = 1,2,...,s). \end{aligned}\right\}$$

Formulas (11) demonstrate that the numerators α_{kj} located on the right-hand side of the equation (9) are expressed in terms of the values of a polynomial $r(\lambda)$ situated on the matrix spectrum A. The following values are defined as being equal to the corresponding values of the function $f(\lambda)$ and its derivatives. Thus,

$$\alpha_{k_1} = \frac{f(\lambda_k)}{\psi_k(\lambda_k)},$$

$$\alpha_{k_2} = f(\lambda_k)\left[\frac{1}{\psi_k(\lambda)}\right]'_{\lambda=\lambda_k} + f'(\lambda_k)\frac{1}{\psi_k(\lambda_k)} ...(k = 1,2,...,s). \tag{12}$$

The following is the shorthand notation for the formulas (12):

$$\alpha_{kj} = \frac{1}{(i-1)!}\left[\frac{f(\lambda)}{\psi_k(\lambda)}\right]^{(j-1)}_{\lambda=\lambda_k} \quad (j = 1,2,...,m_k;\ k = 1,2,...,s). \tag{13}$$

Subsequent to the determination of all α_{kj}, the $r(\lambda)$、 are ascertained through the utilisation of the formula. This formula follows from multiplying both sides of the equation (9) by $\psi(\lambda)$:

$$r(\lambda)=\sum_{k=1}^{s}\left[\alpha_{k_1}+\alpha_{k_2}(\lambda-\lambda_k)+...+\alpha_{k,m_k}(\lambda-\lambda_k)^{m_k-1}\right]\cdot\psi_k(\lambda). \qquad (14)$$

In expression (14), the term in square brackets—the coefficient of $\psi_k(\lambda)$, in the (13) power can be expressed as the sum of the first m_k terms of the Taylor series expansion $(\lambda-\lambda_k)$ for the function $\frac{f(\lambda)}{\psi_k(\lambda)}$.

Let be an A real matrix. It can thus be demonstrated that the minimal polynomial $\psi(\lambda)$ has real coefficients and roots. The characteristic numbers λ_i, may be real or complex conjugate in pairs; in the case of $\lambda_g=\overline{\lambda_h}$, the multiplicities $m_g=m_h$. are equal. In the context of a real function $f(\lambda)$ on the spectrum of matrix A it can be demonstrated that all values of λ_i must be real. Furthermore, in the case of two complex-conjugate eigenvalues λ_h and $\lambda_g=\overline{\lambda_h}$, it is evident that the corresponding values on the spectrum are complex $f(\lambda_g)=\overline{f(\lambda_h)}, f'(\lambda_g)=\overline{f'(\lambda_h)},...$ It can thus be demonstrated that, by virtue of formula (14), the interpolation polynomial $r_A(\lambda)$ has real coefficients. However, $r(A)$, and consequently $f(A)=r(A)$, constitutes a genuine matrix [3-4].

To illustrate this point, consider the following example: The aim of this study is to calculate the value of the given function $f(A)=r(A)$on the matrix spectrum:

$$A=\begin{pmatrix}5 & -3 & 2\\ 6 & -4 & 4\\ 4 & -4 & 5\end{pmatrix}$$

Solution. The following investigation will concern the construction of the characteristic polynomial of the matrix A:

$$\Delta(\lambda)=|A-\lambda E|=\begin{vmatrix}5-\lambda & -3 & 2\\ 6 & -4-\lambda & 4\\ 4 & -4 & 5-\lambda\end{vmatrix}==6-11\lambda+6\lambda^2-\lambda^3=(\lambda-1)\cdot(\lambda-2)\cdot(\lambda-3)$$

In order to facilitate comprehension, the polynomial is presented below, along with the simple roots $\lambda_1=1$, $\lambda_2=2$, $\lambda_3=3$. In this particular instance, it is more expedient to adopt the Lagrange–Sylvester interpolation polynomial as the matrix function:

$$P_3(\lambda)=\sum_{i=0}^{3}f(\lambda_i)\varphi_i(x)=f(\lambda_1)\frac{(\lambda-2)\cdot(\lambda-3)}{(1-2)\cdot(1-3)}+f(\lambda_2)\frac{(\lambda-1)\cdot(\lambda-3)}{(2-1)\cdot(2-3)}+f(\lambda_3)\frac{(\lambda-1)\cdot(\lambda-2)}{(3-1)\cdot(3-2)}=$$
$$=\frac{1}{2}(\lambda-2)\cdot(\lambda-3)f(1)-(\lambda-1)\cdot(\lambda-3)\cdot f(2)+\frac{1}{2}(\lambda-1)\cdot(\lambda-2)\cdot f(3)$$

In consideration of the specified function $f(\lambda)$, which is defined on the spectrum of the matrix A, the following results are obtained:

$$r_A(\lambda)=\frac{1}{2}(\lambda-2)\cdot(\lambda-3)\cdot f(1)-(\lambda-1)\cdot(\lambda-3)\cdot f(2)+\frac{1}{2}(\lambda-1)\cdot(\lambda-2)\cdot f(3) \quad (15)$$

For the purpose of this study, the function $f(A)=\ln A$ will be located, then, for this function $r_A(\lambda)$ when $\lambda=A$ the formula (15) looks like:

$$\ln A=r(A)=\frac{1}{2}(A-2E)\cdot(A-3E)\cdot\ln 1-(A-E)\cdot(A-3E)\cdot\ln 2+\frac{1}{2}(A-E)\cdot(A-2E)\cdot\ln 3$$

Substituting the matrix A and the identity matrix E, we obtain the value of the matrix function:

$$\ln A=r(A)=-\begin{pmatrix}4&-3&2\\6&-5&4\\4&-4&4\end{pmatrix}\cdot\begin{pmatrix}2&-3&2\\6&-7&4\\4&-4&2\end{pmatrix}\cdot\ln 2+\frac{1}{2}\begin{pmatrix}4&-3&2\\6&-5&4\\4&-4&4\end{pmatrix}\cdot\begin{pmatrix}3&-3&2\\6&-6&4\\4&-4&3\end{pmatrix}\cdot\ln 3=$$

$$=\begin{pmatrix}-2\ln 2+4\ln 3&\ln 2-4\ln 3&4\ln 3\\-2\ln 2+8\ln 3&\ln 2-8\ln 3&8\ln 3\\8\ln 3&-8\ln 3&8\ln 3\end{pmatrix}$$

As we can see, using formula (15), we can find the value of any function with a matrix argument. Given the computational complexity of the process, we will perform the calculation using the modern computer software package Maple. We will use the specialised LinearAlgebra package, which is designed for matrix operations. The initial matrix is entered, and the characteristic and minimal polynomials are computed. The roots of the minimal polynomial are then found, and subsequently factored [5]:

```
restart;
with (LinearAlgebra);
A:=Matrix(3,3,[5,-3,2,6,-4,4,4,-4,5]);
Xm:=factor((CharacteristicMatrix(A,lambda)));
Mm:=factor((MinimalPolynomial(A,lambda)));
KMm:=solve(Mm,lambda);
lambda1:=KMm[1];lambda2:=KMm[2];lambda3:=KMm[3];
```

$$A:=\begin{bmatrix}5&-3&2\\6&-4&4\\4&-4&5\end{bmatrix}$$

$$Xm:=\begin{bmatrix}\lambda-5&3&-2\\-6&\lambda+4&-4\\-4&4&\lambda-5\end{bmatrix}$$

$$Mm := (\lambda - 1)(\lambda - 2)(\lambda - 3)$$

$$KMm := 1, 2, 3$$

$$\lambda 1 := 1 \quad \lambda 2 := 2 \quad \lambda 3 := 3$$

Evidently, the roots of the minimal polynomial are distinct, as would be expected according to the theory. The formula for the interpolation polynomial is therefore written down as follows:

```
FA:=f(lambda1)*P1+f(lambda2)*P2+f(lambda3)*P3;
```

$$FA := f(1)\, P1 + f(2)\, P2 + f(3)\, P3$$

Let us calculate the polynomials in turn $P1, P2, P3$, using the symbols for each characteristic number, and then we derive the formula again FA, which, in this case, we shall denote as $FA1$ [6]:

```
P1:=((lambda-lambda2)*(lambda-lambda3))/((lambda1-lambda2)*(lambda1-lambda3));
P2:=((lambda-lambda1)*(lambda-lambda3))/((lambda2-lambda1)*(lambda2-lambda3));
P3:=((lambda-lambda1)*(lambda-lambda2))/((lambda3-lambda1)*(lambda3-lambda2));
FA1:=f(lambda1)*P1+f(lambda2)*P2+f(lambda3)*P3;
```

$$P1 := \frac{(\lambda - 2)(\lambda - 3)}{2}$$

$$P2 := -(\lambda - 3)(\lambda - 1)$$

$$P3 := \frac{(\lambda - 1)(\lambda - 2)}{2}$$

$$FA1 := \frac{1}{2} f(1)(\lambda - 2)(\lambda - 3) - f(2)(\lambda - 3)(\lambda - 1) + \frac{1}{2} f(3)(\lambda - 1)(\lambda - 2)$$

Given a function expressed as λ in $FA1$, we convert it into an expression containing matrix elements. To do this, we work with the numerator of each polynomial $P1, P2, P3$, at the same time λ replace with a matrix A, and we replace the characteristic numbers with the product of that number and the identity matrix E [5]-[6]:

```
A1:=MatrixAdd(A,Multiply(IdentityMatrix(3),(-lambda1)));
A2:=MatrixAdd(A,Multiply(IdentityMatrix(3),(-lambda2)));
A3:=MatrixAdd(A,Multiply(IdentityMatrix(3),(-lambda3)));
ChP1A:=Multiply(A2,A3);
ChP2A:=Multiply(A1,A3);
ChP3A:=Multiply(A1,A2);
```

$$A1 := \begin{bmatrix} 4 & -3 & 2 \\ 6 & -5 & 4 \\ 4 & -4 & 4 \end{bmatrix} \quad A2 := \begin{bmatrix} 3 & -3 & 2 \\ 6 & -6 & 4 \\ 4 & -4 & 3 \end{bmatrix} \quad A3 := \begin{bmatrix} 2 & -3 & 2 \\ 6 & -7 & 4 \\ 4 & -4 & 2 \end{bmatrix}$$

$$ChP1A := \begin{bmatrix} -4 & 4 & -2 \\ -8 & 8 & -4 \\ -4 & 4 & -2 \end{bmatrix} \quad ChP2A := \begin{bmatrix} -2 & 1 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ChP3A := \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 4 & -4 & 4 \end{bmatrix}$$

All numerators $P1, P2, P3$, As expected, they are represented as matrices. For the matrix representation $P1, P2, P3$ all that remains is to multiply the numerator and denominator of the polynomial:

```
PP1:=Multiply(ChP1A,denom(P1));
PP2:=Multiply(ChP2A,denom(P2));
PP3:=Multiply(ChP3A,denom(P3));
```

$$PP1 := \begin{bmatrix} -8 & 8 & -4 \\ -16 & 16 & -8 \\ -8 & 8 & -4 \end{bmatrix} \quad PP2 := \begin{bmatrix} -2 & 1 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad PP3 := \begin{bmatrix} 4 & -4 & 4 \\ 8 & -8 & 8 \\ 8 & -8 & 8 \end{bmatrix}$$

All the polynomials that make up the formula $FA1$, are represented using matrices. The formula for calculating values with a matrix argument is now ready. Let's define the function $f(\lambda) = \ln\lambda$ for calculation $f(A) = \ln A$. And we write the commands to calculate the values $f(1), f2, f(3)$, using the expression simplification command *simplify* [6]:

```
ff:=ln(lambda);
ff1:=simplify(subs(lambda=lambda1,ff));
ff2:=simplify(subs(lambda=lambda2,ff));
ff3:=simplify(subs(lambda=lambda3,ff));
```

$$ff := \ln(\lambda) \quad ff1 := 0 \quad ff2 := \ln(2) \quad ff3 := \ln(3)$$

Let's calculate each term FA and sum them using the command *simplify* [6]:

```
PP1ff1:=simplify(Multiply(PP1,ff1));
PP2ff2:=simplify(Multiply(PP2,ff2));
PP3ff3:=simplify(Multiply(PP3,ff3));
FA4:=PP1ff1+PP2ff2+PP3ff3;
```

$$PP1ff1 := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad PP2ff2 := \begin{bmatrix} -2\ln(2) & \ln(2) & 0 \\ -2\ln(2) & \ln(2) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$PP3ff3 := \begin{bmatrix} 4\ln(3) & -4\ln(3) & 4\ln(3) \\ 8\ln(3) & -8\ln(3) & 8\ln(3) \\ 8\ln(3) & -8\ln(3) & 8\ln(3) \end{bmatrix}$$

$$FA4 := \begin{bmatrix} -2\ln(2)+4\ln(3) & \ln(2)-4\ln(3) & 4\ln(3) \\ -2\ln(2)+8\ln(3) & \ln(2)-8\ln(3) & 8\ln(3) \\ 8\ln(3) & -8\ln(3) & 8\ln(3) \end{bmatrix}$$

Functions with matrix arguments represent a mathematical generalization of functions of numerical arguments. Of particular interest is the determination of functions of matrices and the calculation of their values, primarily in connection with the solution of applied problems. It is possible to minimize the computational complexity – which is related not only to the increase in the order of the matrix but also to the presence of simple and multiple roots of the matrix's characteristic polynomial – by performing such computations in modern computer mathematics software packages. This article develops code for computing a function with a matrix argument, in the case of simple roots of the matrix's characteristic polynomial, in the Maple system.

```
restart;
with (LinearAlgebra);
a11:=5:a12:=-3:a13:=2:a21:=6:a22:=-4:a23:=4:
a31:=4:a32:=-4:a33:=5:
A:=Matrix(3,3,[a11,a12,a13,a21,a22,a23,a31,a32,a33]);
```

Research findings. Currently, great importance is attached to methods that can provide efficient solutions, characterised by high computational speed and avoiding complex calculations. The present study has sought to contribute to the existing body of knowledge on the subject through the conduction of empirical testing. The results of this testing have confirmed the hypothesis that, in the case of simple roots of the characteristic polynomial, it is more rational to find functions of a matrix argument using the Lagrange–Sylvester interpolation polynomial. Efficiency is improved when performing calculations in a computer algebra system, which suggests the use of automated code not only for matrices of order 3, but also of higher order: for this, it is necessary to add polynomials Pi sequentially.

Discussion of the research results. The practical implementation of this system within the domain of computer mathematics facilitates the examination of the evaluation of a function of matrices in the case of multiple roots of the matrix's characteristic polynomial. The code developed will undoubtedly complement the automated code presented in this article.

Conclusion. The application of computer technologies facilitates the implementation and operation of methods which, to date, combine the implementation of mathematical methods in computer mathematics systems whilst taking into account their specific characteristics. The developed codes form the basis for widespread use in various applications.

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ВЫЧИСЛЕНИЕ ФУНКЦИЙ С МАТРИЧНЫМ АРГУМЕНТОМ В СИСТЕМЕ КОМПЬЮТЕРНОЙ МАТЕМАТИКИ

Аннотация.Функции с матричным аргументом представляют собой математическое расширение функций от числовых аргументов. Интерес представляет нахождение функций от матриц и вычисление их значений прежде всего связанный с решением прикладных задач. Минимизировать трудоемкость вычислительного процесса, который связан не только с увеличением порядка матрицы, но и наличием простых и кратных корней характеристического полинома матрицы возможно при проведении таких вычислений в современных пакетах компьютерной математики. В статье разрабатывается код вычисления функции с матричным аргументом, в случае простых корней характеристического полинома матрицы в системе Maple.

Ключевые слова: матричный аргумент, характеристические корни, интерполяционный полином Лагранжа - Сильвестра, спектр матрицы, минимальный полином

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КОМПЬЮТЕРЛІК МАТЕМАТИКА ЖҮЙЕСІНДЕ МАТРИЦАЛЫҚ АРГУМЕНТІ БАР ФУНКЦИЯЛАРДЫ ЕСЕПТЕУ

Аннотация.Матрицалық аргументі бар функциялар сандық аргументтерден функциялардың математикалық кеңеюін сипаттайды. Матрицалардан функцияларды табу және олардың мәндерін есептеу ең алдымен қолданбалы есептерді шешумен байланысты қызығушылық тудырады. Компьютерлік математиканың қазіргі заманғы пакеттерінде осындай есептеулерді жүргізу кезінде матрица тәртібінің ұлғаюымен ғана емес, сондай-ақ матрицаның сипаттамалық көпмүшелігінің қарапайым және еселі түбірлерінің болуымен байланысты есептеу процесінің еңбек сыйымдылығын барынша азайтуға болады. Мақалада Maple жүйесіндегі сипаттамалық матрица көпмүшелігінің қарапайым түбірлері жағдайында матрицалық дәлелі бар функцияны есептеу коды әзірленеді.

Кілт сөздер: матрицалық аргумент, сипаттамалық түбірлер, Лагранж - Сильвестр интерполяциялық көпмүшеліктері, матрица спектрі, минималды көпмүшелік